A model for simulating gas bubble entrainment in two-phase horizontal slug flow: a discussion

Daniel Cruz

Introduction

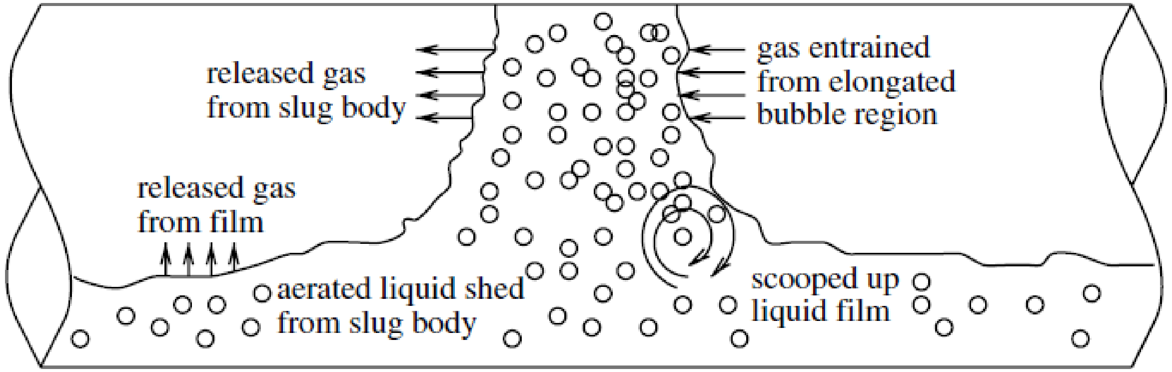

Fig. 1. Aerated slug unit.

In slug flow, gas is often entrained from the large elongated gas (often called Taylor) bubble into the liquid slug and this is thought (if not taken for granted) to have a significant effect on the slug behaviour. It is hence desirable to model this phenomenon in order to understand its importance as well as to improve the accuracy of the prediction of slug characteristics. The present study is focused on the modelling of this gas bubble entrainment (also referred to as ‘‘aeration’’) and its effects on liquid slugs in horizontal gas–liquid flow in pipes.

Introduction

The most common approach for incorporating gas entrainment in slug flow calculations is the use of empirical correlations for the dispersed bubble void fraction in the slug. They correlated their results in terms of the mixture velocity UM only as:

Gregory et al. (1978)

$$\alpha_{Ls} = \frac{1}{1 + (U_M/8.66)^{1.39}},$$

where, in the slug body, the fundamental relation:

$$\alpha_B = 1 - \alpha_{Ls}$$

$$\alpha_{Ls} = 1 - \frac{U_M}{C_c + U_M},$$

Malnes (1982) where C_c is a dimensional coefficient defined as:

$$C_c = 83\left(\frac{g\sigma}{\rho_L}\right)^{0.25},$$

Jepson (1987).

$$\alpha_{Ls} = 1 - \frac{U_M - u_{mf}}{U_M + u_{m0}}$$

with

$$u_{mf} = 2.6\left[1 - 2\left(\frac{D_0}{D}\right)^2\right]\sqrt{gD}, \quad D_0 = 2.5 \text{ cm},$$

$$u_{m0} = 2400\left[1 - \frac{1}{3}\sin\beta\right]Bo^{-3/4}\sqrt{gD},$$

$$Bo = \frac{\rho_L g D^2}{\sigma}.$$

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It is easy to show (Bonizzi, 2003) that the liquid and centre of mass velocities are practically identical in a fully dispersed flow with high liquid to gas density ratios. Thus, the liquid and bubble velocities may be expressed as:

$$u_L \approx u_M \qquad u_B \approx u_M + u_s,$$

In the above equations, the subscripts L, B, and M relate to liquid, gas bubble, and the mixture respectively, while u_s represents the slip velocity between the gas bubbles and liquid:

$$u_s = u_B - u_L.$$

The following quantities are now defined: a_L is the volume fraction of the liquid component (unaerated) at any point, a_G is the volume fraction of the gas phase flowing separately, i.e. in the stratified region, a_B is the volume fraction of the gas bubbles entrained in the slug body, and a_M stands for the volume fraction of the mixture of liquid and dispersed (i.e. entrained) gas bubbles. The compatibility equation which needs to be enforced therefore is that between a_M and a_G and is given by:

$$\alpha_M + \alpha_G = 1. \qquad \alpha_M = \alpha_B + \alpha_L. \quad \rho_M = (1 - \alpha_B)\rho_L + \alpha_B \rho_G.$$

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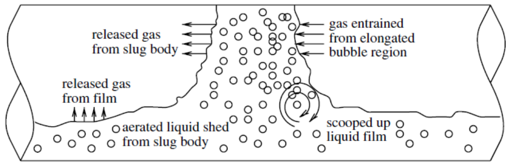

Fig. 1. Aerated slug unit.

When the above assumptions are introduced, the governing equations for an isothermal transient one-dimensional stratified and aerated slug flow become:

- gas continuity equation:

$$\frac{\partial(\rho_G \alpha_G)}{\partial t} + \frac{\partial(\rho_G \alpha_G u_G)}{\partial x} = -\dot{m}_B,$$

- mixture continuity equation:

$$\frac{\partial(\rho_M \alpha_M)}{\partial t} + \frac{\partial(\rho_M \alpha_M u_M)}{\partial x} = \dot{m}_B,$$

- gas momentum equation:

$$\frac{\partial(\rho_G \alpha_G u_G)}{\partial t} + \frac{\partial(\rho_G \alpha_G u_G^2)}{\partial x} = -\alpha_G \frac{\partial p}{\partial x} + \rho_G \alpha_G g \sin\beta + F_{wG} + F_i,$$

- mixture momentum equation:

$$\frac{\partial(\rho_M \alpha_M u_M)}{\partial t} + \frac{\partial(\rho_M \alpha_M u_M^2)}{\partial x} = -\alpha_M \frac{\partial p}{\partial x} - \rho_M \alpha_M g \frac{\partial h}{\partial x} \cos\beta + \rho_M \alpha_M g \sin\beta + F_{wL} - F_i,$$

Need a model

A scalar-transport equation for the conservation o mass of the gas bubbles entrained within the liquid slug can be formulated from mass conservation considerations as:

$$\frac{\partial(\rho_G \alpha_B)}{\partial t} + \frac{\partial(\rho_G \alpha_B u_B)}{\partial x} = \dot{m}_B.$$

The friction factors are based on the widely used model of Taitel and Dukler (1976).
The expression for the gas-wall friction factor is:

$$f_G = C_G Re_G^{-n_G},$$

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Bubble velocity

The bubble velocity in Eq. (20) needs to be specified and a closure model is required for that purpose; it is here where a drift-flux type relationship is introduced.

$$u_B = u_M - \sqrt{\frac{-4d_B(dp/dx)}{3C_D\rho_L}}.$$

I am not sure about this, but…

Mass exchange rates

The shedding rate of dispersed bubbles at the slug tail is obtained by assuming *that all the bubbles arriving at the tail leave the slug and enter the large gas bubble behind it*. Hence:

$$\dot{M}_B = \rho_G A(u_b - u_B)\alpha_B,$$

where u_b represents the local velocity at which the tail of the slug propagates (see Fig. 4) and whose value is determined as explained later. *The entrainment rate at the slug front must however be obtained from an independent corre*

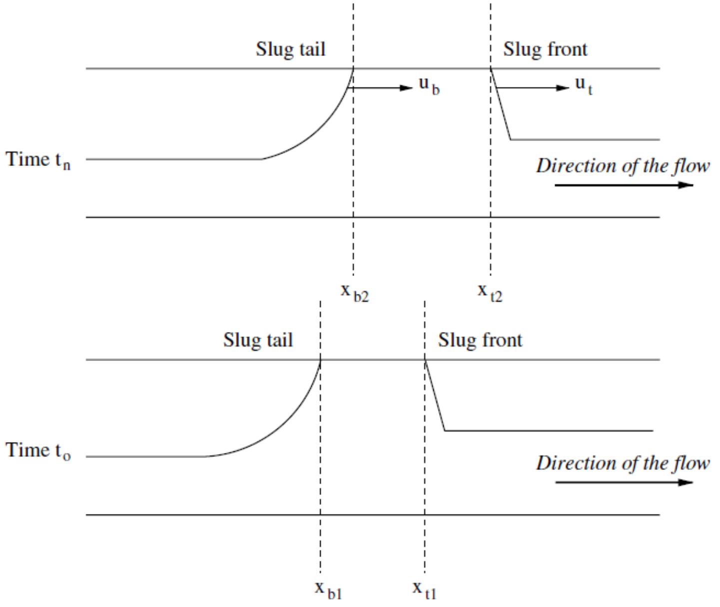

Fig. 4. Displacement of front and tail of the slug as it travels along the pipe.

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Not many such correlations are available in the literature, and only three could be found.

$$\dot{M}_B = \rho_G A\left(0.076\frac{S_i}{D}(u_t - u_{Lf}) - 0.15\right),$$

$$\dot{M}_B = 1.871\rho_G A S_i[(u_t - u_{Lf}) - 2.126].$$

$$\dot{M}_B = \rho_G A_{Lf}(u_t - u_{Lf})\zeta(Fr - 1)^{\varepsilon},$$

where u_t represents the velocity of propagation of the front of the slug (see Fig. 4), *u_{Lf} the velocity of the liquid in the film region*, and *S_i the gas–liquid interfacial width in the film.*

My suggestion: $\delta_b \rightarrow$ *Bubble radius*; $H \rightarrow$ *Slug Length(??)*,

$$\dot{M}_B = C\rho_g \frac{u_t - u_{Lf}}{H} A\alpha_g\delta_b$$

$$\delta b = H\frac{Wec}{We}\frac{1}{f}$$

$$We \equiv \frac{\rho U^2 H}{\sigma}$$

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Film and slug front/tail velocities

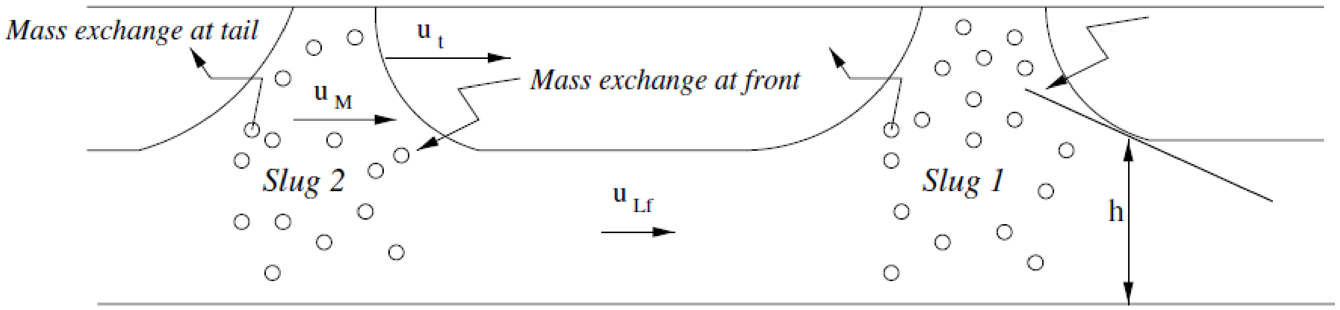

Fig. 5. Quantities needed to calculate the entrainment rate.

It is assumed herein that the *front and tail of a slug travel at the same velocity*. Hence, both slug velocities are based on the following equation:

$$u_b = C_0 u_M + u_d$$

The alternative method for determining these velocities is to continually track the movements of each slug front and tail in time and then calculate the velocities simply from the displacement of these points over successive time steps in the numerical integration.

$$u_t = \frac{x_{t2} - x_{t1}}{t_n - t_0}, \qquad u_b = \frac{x_{b2} - x_{b1}}{t_n - t_0},$$

with the distribution parameter C_0 and drift velocity u_d being 1.2 and 0, or 1.05 and $0.54\sqrt{gD}$ depending on whether the mixture Froude number ($Fr_M = U_M/(\sqrt{gD})$) is greater or smaller than the critical value of 3.5 respectively. For a horizontal pipe with internal diameter of 0.078 mm, the Bendiksen correlation reduces to:

$$u_b = 1.2u_M \qquad (59)$$

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<u>Some discussion points:</u>

The model assumes that the aeration will always occur, no matter the slug or gas velocities. In my opinion there will be cases that even though the gas is captured by the slug, it will boil up and will not aerate the slug.

$$C_D \frac{ut^2}{2} A_b \rho_L > g\, Vol_b\, \rho_L$$

Using the bubble radius definition, we have that the condition for the bubble to be trapped is:

$$f > \sqrt{2}\sqrt{\frac{We_c}{We}}\frac{1}{Fr}$$

<u>This equation means that only for values of the friction factor higher than some specific limit, the aeration process is effective!?</u>