Slug Capturing
(solver)

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Introduction

3 possible ways to simulate slug flows in pipes:

1. "Unit-Cell" approach:
   a. Simple and fast
   b. (Quasi) steady-state analysis
   c. Unable to predict transition between flow patterns

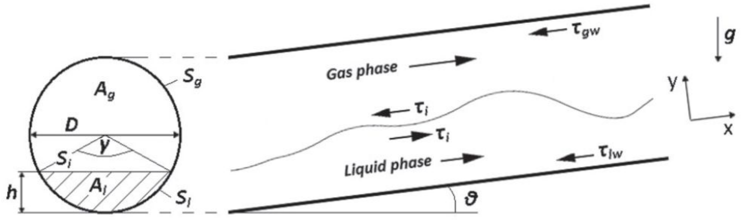
2. "Slug tracking" method:
   a. Each slug tracks individually
   b. Can compute Slug characteristics
   c. Unable to simulate the transition between flow patterns

3. "Slug capturing" method:
   a. Can compute Slug characteristics
   b. Slug formation, growth, and decay arise naturally from the numerical solution
   c. Slower than 2 former methods

Governing Equations

- 4-equation compressible 2-fluid:

$$\frac{\partial(\alpha_g\rho_g)}{\partial t}+\frac{\partial(\alpha_g\rho_g u_g)}{\partial x}=0 \qquad \frac{\partial(\alpha_l\rho_l)}{\partial t}+\frac{\partial(\alpha_l\rho_l u_l)}{\partial x}=0$$



$$\frac{\partial(\alpha_g\rho_g u_g)}{\partial t}+\frac{\partial(\alpha_g\rho_g u_g^2)}{\partial x}+\frac{\partial(\alpha_g p_g)}{\partial x}-p_{ig}\frac{\partial\alpha_g}{\partial x}=-\rho_g\alpha_g g\sin(\theta)-F_{gw}-F_i$$

$$\frac{\partial(\alpha_l\rho_l u_l)}{\partial t}+\frac{\partial(\alpha_l\rho_l u_l^2)}{\partial x}+\frac{\partial(\alpha_l p_l)}{\partial x}-p_{il}\frac{\partial\alpha_l}{\partial x}=-\rho_l\alpha_l g\sin(\theta)-F_{lw}+F_i.$$

$$\alpha_g+\alpha_l=1$$

- Pressure $\frac{\partial(\alpha_k p_k)}{\partial x}-p_{ik}\frac{\partial\alpha_k}{\partial x}=\alpha_k\frac{\partial p_{ik}}{\partial x}+\rho_k\alpha_k g\frac{\partial h}{\partial x}\cos(\theta),$
  - where EOS $p_{ik}=c_k^2(\rho_k-\rho_{k,0})+p_0$ & $\frac{\partial h}{\partial x}\cos\theta=\zeta\frac{\partial\alpha_g}{\partial x}$ in which $\zeta=-\frac{\pi D\cos\theta}{4\sin(\gamma/2)}$

- Issue : 4 equations and 5 unknowns ⇨ λ is not always real ⇨ system is ill-posed.

- To obtain well-posedness, 5-equation model is used which is strictly hyperbolic.

$$\frac{\partial\alpha_g}{\partial t}+u_i\frac{\partial\alpha_g}{\partial x}=0 \quad \Longrightarrow \quad u_i=\frac{\sum\alpha_k\rho_k u_k}{\sum\alpha_k\rho_k}$$

5-equation system

❖ Quasi-linear form of non-linear System Of Equations (SOE) using ROE method:

$$\frac{\partial \mathbf{q}}{\partial t} + \mathbf{A}(\mathbf{q})\frac{\partial \mathbf{q}}{\partial x} = \mathbf{s}(\mathbf{q})$$

➢ vector of Composite variables

$$\mathbf{q} = \left[\alpha_g \;\; \alpha_g\rho_g \;\; \alpha_g\rho_g u_g \;\; \alpha_l\rho_l \;\; \alpha_l\rho_l u_l\right]^T$$

➢ Source term vector

$$\mathbf{s}(\mathbf{q}) = \left[0 \;\; 0 \;\; -\alpha_g\rho_g g - F_{gw} - F_{\mathrm{i}} \;\; 0 \;\; -\alpha_l\rho_l g - F_{lw} + F_{\mathrm{i}}\right]^T$$

➢ matrix

$$\mathbf{A} = \begin{bmatrix} u_{\mathrm{i}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\rho_g c_g^2 - \alpha_g\rho_g g\zeta & c_g^2 - u_g^2 & 2u_g & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \rho_l c_l^2 - \alpha_l\rho_l g\zeta & 0 & 0 & c_l^2 - u_l^2 & 2u_l \end{bmatrix}$$

❖ Eigenvalues & Right eigenvectors of matrix $\mathbf{A}$

$$\lambda = \left[u_{\mathrm{i}} \;\; u_g - c_g \;\; u_g + c_g \;\; u_l - c_l \;\; u_l + c_l\right]$$

&

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{c_g^2\rho_g + \alpha_g\rho_g g\zeta}{\left[(u_g - u_{\mathrm{i}})^2 - c_g^2\right]} & 1 & 1 & 0 & 0 \\ -\frac{c_g^2\rho_g + \alpha_g\rho_g g\zeta}{\left[(u_g - u_{\mathrm{i}})^2 - c_g^2\right]}u_{\mathrm{i}} & u_g - c_g & u_g + c_g & 0 & 0 \\ \frac{c_l^2\rho_l - \alpha_l\rho_l g\zeta}{\left[(u_l - u_{\mathrm{i}})^2 - c_l^2\right]} & 0 & 0 & 1 & 1 \\ \frac{c_l^2\rho_l - \alpha_l\rho_l g\zeta}{\left[(u_l - u_{\mathrm{i}})^2 - c_l^2\right]}u_{\mathrm{i}} & 0 & 0 & u_l - c_l & u_l + c_l \end{bmatrix}$$

Closure models

- Closure relations for liquid-wall, gas-wall and interfacial shear forces in source term:

$$F_{lw} = \frac{\tau_{lw} S_l}{A}, \qquad F_{gw} = \frac{\tau_{gw} S_g}{A}, \qquad F_i = \frac{\tau_i S_i}{A},$$

$$\tau_{lw} = \frac{1}{2} f_{lw} \rho_l |u_l| u_l, \qquad \tau_{gw} = \frac{1}{2} f_{gw} \rho_g |u_g| u_g, \qquad \tau_i = \frac{1}{2} f_i \rho_g |u_g - u_l| (u_g - u_l)$$

- Gas-wall and interfacial friction factors:

$$f_g = \begin{cases} \dfrac{16}{Re_g} & \text{if } Re_g < 2100, \\ 0.046 (Re_g)^{-0.2} & \text{if } Re_g \geqslant 2100; \end{cases} \qquad f_i = \begin{cases} \dfrac{16}{Re_i} & \text{if } Re_i < 2100, \\ 0.046 (Re_i)^{-0.2} & \text{if } Re_i \geqslant 2100. \end{cases}$$

- Liquid-wall friction factor:

$$f_l = \begin{cases} \dfrac{24}{Re_l} & \text{if } Re_l < 2100, \\ 0.0262 (\alpha_l Re_{sl})^{-0.139} & \text{if } Re_l \geqslant 2100. \end{cases}$$

- Reynolds numbers:

$$Re_g = \frac{4 A_g u_g \rho_g}{(S_g + S_\text{i}) \mu_g}, \qquad Re_i = \frac{4 A_g |u_g - u_l| \rho_g}{(S_g + S_\text{i}) \mu_g},$$

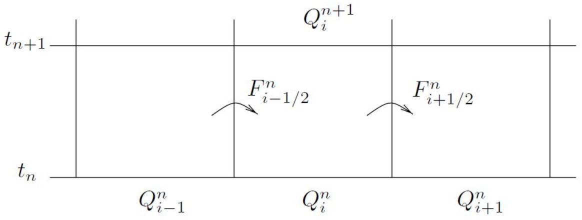
$$Re_l = \frac{4 A_l u_l \rho_l}{S_l \mu_l}, \qquad Re_{sl} = \frac{D u_{sl} \rho_l}{\mu_l},$$

Numerical methods (1/2)

- Spatial discretization on a uniform, 1d grid & 1st-order explicit for time discretization.

- Solution based on succession of operators in a fractional-step FVM:

$$\mathbf{Q}_i^{n+1} = \mathbf{L}_s^{\Delta t}\mathbf{L}_h^{\Delta t}\mathbf{Q}_i^n$$



1. Hyperbolic operator $\mathbf{L}_h^{\Delta t}$ computed using high-resolution extension of Godunov's method:

$$\mathbf{Q}_i^{h,n+1} = \mathbf{Q}_i^n \underbrace{- \frac{\Delta t}{\Delta x}\left(A^-\Delta\mathbf{Q}_{i+\frac{1}{2}}^n + A^+\Delta\mathbf{Q}_{i-\frac{1}{2}}^n\right) - \frac{\Delta t}{\Delta x}\left(\tilde{\mathbf{F}}_{i+\frac{1}{2}}^n - \tilde{\mathbf{F}}_{i-\frac{1}{2}}^n\right)}_{\mathbf{L}_h^{\Delta t}}$$

2. Operator $\mathbf{L}_s^{\Delta t}$ to add source terms and take into account the pressure relaxation

$$\mathbf{Q}_i^{n+1} = \mathbf{Q}_i^{h,n+1} + \underbrace{\Delta t\mathbf{S}(\mathbf{Q}_i^n)}_{\mathbf{L}_s^{\Delta t}}$$

Numerical methods (2/2)

- *ROE5* scheme is used to solve SOE:
  - Waves from neighbor cells

$$A^{-}\Delta \mathbf{Q}^{n}_{i-\frac{1}{2}} = \sum_{p=1}^{m} \left(\lambda^{p}_{i-\frac{1}{2}}\right)^{-} \mathbf{W}^{p}_{i-\frac{1}{2}},$$

$$A^{+}\Delta \mathbf{Q}^{n}_{i-\frac{1}{2}} = \sum_{p=1}^{m} \left(\lambda^{p}_{i-\frac{1}{2}}\right)^{+} \mathbf{W}^{p}_{i-\frac{1}{2}},$$

$$\mathbf{W}^{p}_{i-\frac{1}{2}} = \beta^{p}_{i-\frac{1}{2}} \mathbf{r}^{p}_{i-\frac{1}{2}}$$

$$\beta_{i-\frac{1}{2}} = \mathbf{R}^{-1}_{i-\frac{1}{2}} (\mathbf{Q}_i - \mathbf{Q}_{i-1})$$

$$\left(\lambda^{p}_{i-\frac{1}{2}}\right)^{\pm} = \frac{1}{2}\left(\lambda^{p}_{i-\frac{1}{2}} \pm |\lambda^{p}_{i-\frac{1}{2}}|\right)$$

  - Flux vector :

$$\tilde{\mathbf{F}}^{n}_{i-\frac{1}{2}} = \frac{1}{2}\sum_{p=1}^{m} |\lambda^{p}_{i-\frac{1}{2}}| \left(1 - \frac{\Delta t}{\Delta x}|\lambda^{p}_{i-\frac{1}{2}}|\right) \phi\left(\psi^{p}_{i-\frac{1}{2}}\right) \mathbf{W}^{p}_{i-\frac{1}{2}}$$

    - in flux limiter $\phi\left(\psi^{p}_{i-\frac{1}{2}}\right)$: $\psi^{p}_{i-\frac{1}{2}} = \dfrac{\mathbf{W}^{p}_{I-\frac{1}{2}} \cdot \mathbf{W}^{p}_{i-\frac{1}{2}}}{\mathbf{W}^{p}_{i-\frac{1}{2}} \cdot \mathbf{W}^{p}_{i-\frac{1}{2}}}$

$$I = \begin{cases} i-1 \text{ if } \lambda^{p}_{i-\frac{1}{2}} \geqslant 0, \\ i+1 \text{ if } \lambda^{p}_{i-\frac{1}{2}} < 0. \end{cases}$$

      - MC high-resolution limiter: $\phi(\psi) = max(0, min((1+\psi)/2), 2, 2\psi),$

- Pressure relaxation:

$$\alpha_l = \frac{-\chi_2 - \sqrt{(\chi_2^2 - 4\chi_1\chi_3)}}{2\chi_1},$$

$$\chi_1 = c_l^2 \rho_l^\circ - c_g^2 \rho_g^\circ - f_{relax},$$

$$\chi_2 = -c_l^2(\alpha_l \rho_l + \rho_l^\circ) + c_g^2(-\alpha_g \rho_g + \rho_g^\circ) + f_{relax},$$

$$\chi_3 = c_l^2 \alpha_l \rho_l.$$

$$f_{relax} = \begin{cases} 0 & \text{if the two phases have the same pressure,} \\ \sigma\kappa & \text{if the surface tension effects are considered.} \end{cases}$$

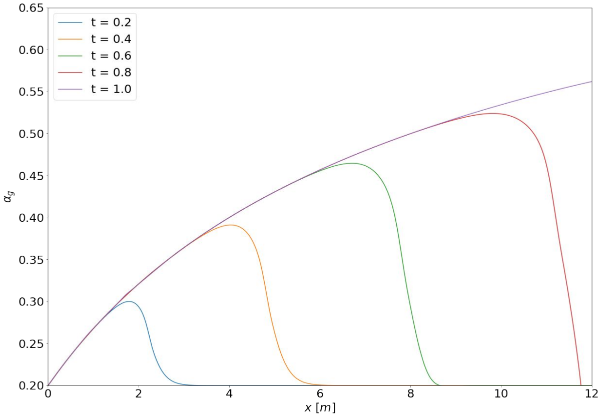
Numerical Test cases

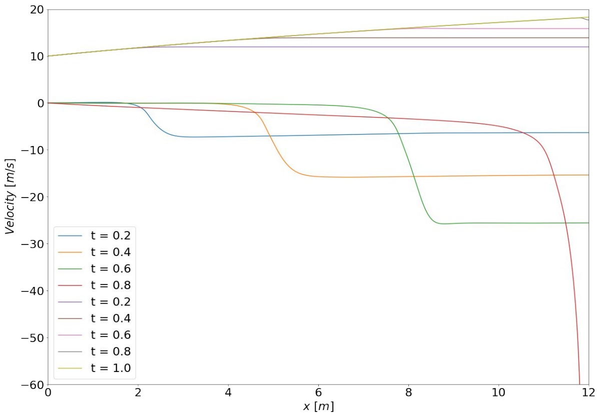
❖ Water Faucet:

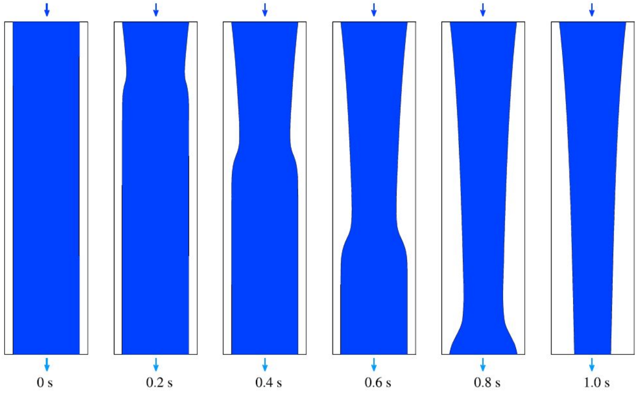
➢ Vertical pipe: $12\text{ m}\ (\theta = 90^\circ)$, $u_l^\circ = 10\text{ m/s}$ $\alpha_l^\circ = 0.8$

➢ Analytical expression:

$$\alpha_g(x,t) = \begin{cases} 1 - \dfrac{\alpha_l^\circ u_l^\circ}{\sqrt{2gx + (u_l^\circ)^2}} & \text{if} \quad x \leqslant u_l^\circ t + \frac{1}{2} g t^2 \\ 1 - \alpha_l^\circ & \text{otherwise,} \end{cases} \qquad u_l(x,t) = \begin{cases} \sqrt{2gx + (u_l^\circ)^2} & \text{if} \quad x \leqslant u_l^\circ t + \frac{1}{2} g t^2 \\ (u_l^\circ)^2 + gt & \text{otherwise,} \end{cases}$$







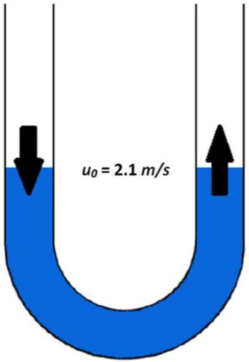
Numerical Test cases

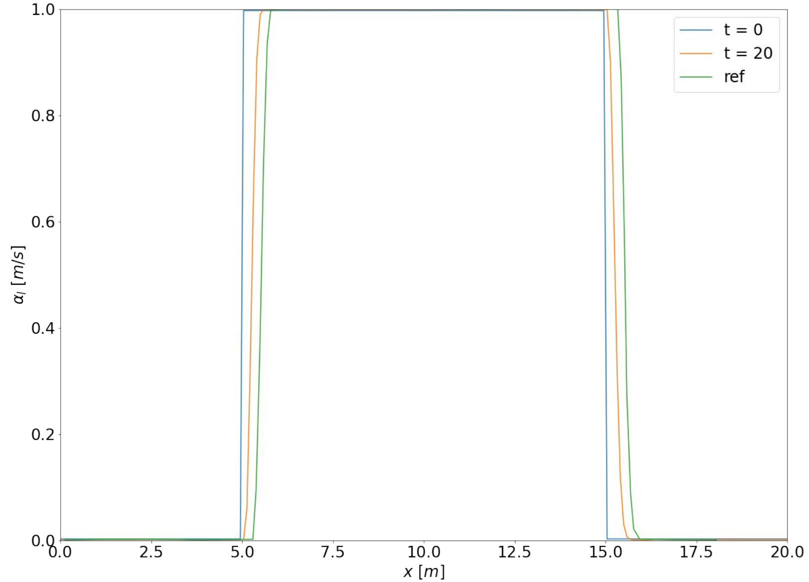
❖ Oscillating Manometer:

➢ U-shape pipe $20\ \mathrm{m}$ with initial condition as:

$$\begin{bmatrix} \alpha_g \\ u_g \\ u_l \\ p \end{bmatrix}_{(x,0)} = \begin{cases} (0.999, 2.1, 2.1, 10^5)^T & \text{if } 0 \leqslant x < 5, \\ \left(0.001, 2.1, 2.1, 10^5 + \rho_{l,0} g \frac{10}{\pi} \sin\left(\frac{\pi(x-5)}{10}\right)\right)^T & \text{if } 5 \leqslant x \leqslant 15, \\ (0.999, 2.1, 2.1, 10^5)^T & \text{if } 15 < x \leqslant 20, \end{cases}$$

$$g_x(x) = \begin{cases} 9.81 & \text{if } 0 \leqslant x < 5, \\ 9.81 \cos\left[\frac{(x-5)}{10}\pi\right] & \text{if } 5 \leqslant x \leqslant 15, \\ 9.81 & \text{if } 15 < x \leqslant 20. \end{cases}$$

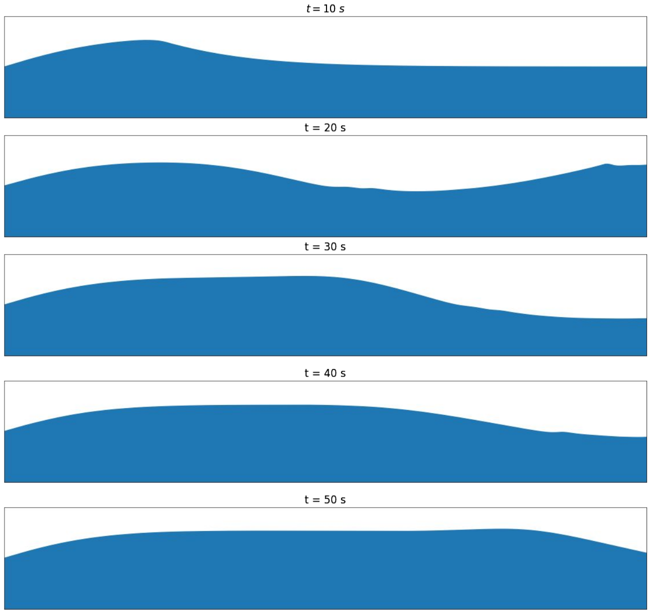


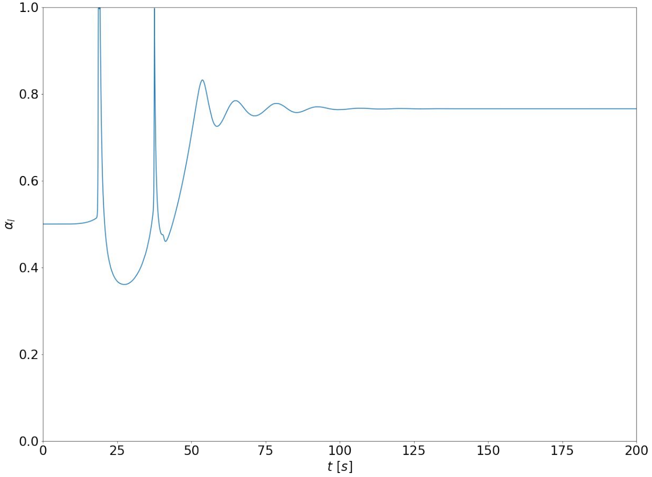


Numerical Test cases

❖ Pipe flow:

- ➢ water-air stratified flow: $u_{sg} = 3.0$ m/s, $u_{sl} = 0.6$ m/s, $\Delta x / D = 0.58$, CFL $= 0.22$
- ➢ Equilibrium liquid volume fraction = 0.766 (Ferrari et al. 2017. = 0.76)



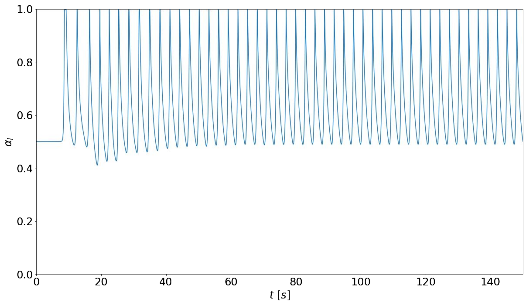


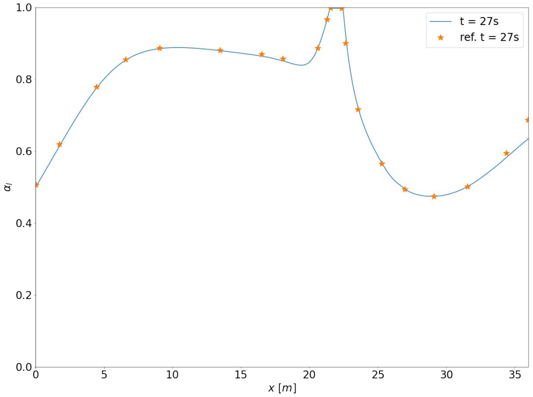
Numerical Test cases

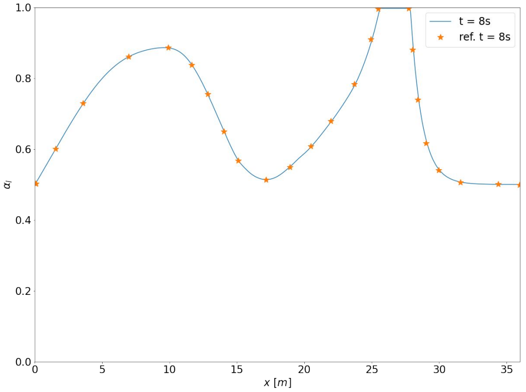
❖ Pipe flow:

➢ water-air slug flow: $u_{sg} = 2.0$ m/s, $u_{sl} = 1.5$ m/s, $\Delta x/D = 0.58$, CFL $= 0.22$

Grid	800	1600	3200
freq (Hz)	0.33	0.33	0.33
ref. freq (Hz)	0.331	-	0.317







Numerical Test cases

❖ Pipe flow:

➢ oil-air slug flow in v-shape pipe in Ferrari et al. 2018: D = 0.0508 m

	c_k $[m/s]$	$\rho_{k,0}$ $[kg/m^3]$	μ_k $[Pa \cdot s]$
air (g)	316	1.0	$1.79 \cdot 10^{-5}$
oil (l)	1000	850.0	0.150

zone 1	zone 2	zone 3
20 m	1 m	20 m
$\theta = 2°$	$\theta = 0°$	$\theta = -2°$

Next month perspective

- ➢ database of case studies (in progress)
- ➢ non-uniform grid (in progress)
- ➢ non-Newtonian
- ➢ vertical pipe (upward flow)